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THE CONSTRUCTION OF A SPATIAL MODEL FOR CALCULATING THE THERMALLY STRESSED STATE OF SHALLOW SHELLS ON A RIGID BASIS

ПОБУДОВА ПРОСТОРОВОЇ МОДЕЛІ ДЛЯ РОЗРАХУНКУ ТЕРМОНАПРУЖЕНОГО СТАНУ ПОЛОГИХ ОБОЛОНОК НА ЖОРСТКІЙ ОСНОВІ

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Abstract: Modern calculations of layered plates and shells in a three-dimensional formulation are based on a technique where the distribution of the desired functions over the thickness of a structure is sought by the method of discrete orthogonalization. In this article, based on the approaches developed by the authors, the thermally stressed state of layered composite shallow shells with a rigidly fixed lower surface is analyzed. The distribution of the desired functions over the thickness of the structure is found based on the exact analytical solution of the system of differential equations. An approach to studying the thermally stressed state of layered composite shells is also considered, and a spatial model for calculating the thermally stressed state of shallow shells on a rigid basis is constructed. Currently, this is a very urgent task when calculating the pavement of bridges. A feature of this approach is the assignment of the desired functions to the outer surfaces of the layers, which allows one to break the layer into sublayers, reducing the approximation error to almost zero.

To build a spatial model, a load option is selected with temperature loads (according to the sine law) and boundary conditions (Navier), which lead to the distribution of the desired functions in terms of a plate with trigonometric harmonics of the Fourier series. A polynomial approximation of the desired functions by thickness is involved.

Using the model under consideration, an analysis of flat layered composite shells on a rigid basis under the influence of temperature load was carried out. The considered example showed that the proposed model provides sufficient accuracy in the calculations of layered shallow shells when considering each layer within one sublayer. When dividing each layer into 32, 64, 128 sublayers, almost the same result was obtained.

The proposed approach can be used as a reference method for testing applied approaches in calculating the stress states of layered shallow composite shells.

Key words: temperature loads, tangential loads, thermo-stressed state, layered composite shells, semi-analytical finite element method.

Introduction. Calculations of layered plates and shells in a three-dimensional setting, including calculations for temperature effects, are most fully presented in [1]. Modern similar calculations are based on a technique where the distribution of unknown functions over the thickness of the structure is sought by the method of discrete orthogonalization.

In this article, based on the approaches developed by the authors, the thermally stressed state of layered composite sloping shells with a rigidly fixed lower surface is analyzed.

1. Construction of the solution

In order to construct a spatial model, the variant of the loading by the temperature load (by the law of the sinus) and the boundary conditions (Navier) is chosen, which leads to the distribution of the desired functions in terms of the plate by the trigonometric harmonics of the Fourier series. The polynomial approximation of the desired functions in thickness is involved.

The layered structure in the Cartesian coordinate system is considered. The faces of the layers are evenly spaced at a certain distance and have zero curvature of the torsion $k_{12} = 0$. The variability of the main curvature is neglected $k_{11} = const$; $k_{22} = const$. The construction is rather slippery, that is, for the coefficients of the first quadratic form should be taken $A_1 \approx 1$; $A_2 \approx 1$. It is assumed that the curvature radii have considerably larger thicknesses of the structure and its outer surfaces have the same curvature, and can also be written $1 + k_{11}z$; $1 + k_{22}z \approx 1$. The introduced restrictions allow us to identify the curvilinear system of orthogonal coordinates with a planar orthogonal coordinate system. The X axis (lower index 1) and Y (lower index 2) are directed in the plan view, Z axis (lower index 3) for its thickness down; a_{k-1}, a_k – the coordinates of the outer surfaces of the k-th layer of the plate along the Z-axis; comma at the level of the index means a differentiation operation.

A well-known approximation of the desired displacements by thickness is used [2]:

$$U_i^{(k)}(x,y,z) = U_{il}^{(k)}(x,y)\, f_{il}^{(k)}(z) + W_{p,i}^{(k)}(x,y)\,\varphi_{2p}^{(k)}(z):$$

$$U_3^{(k)}(x,y,z) = W_p^{(k)}(x,y)\,\beta_p^{(k)}(z), (i = 1,2\,;\ l = 1,2\,;\ p = 1 \ldots 4). \qquad (1)$$

We have $U_{i1}^{(k)}(x,y)$, $U_{i2}^{(k)}(x,y)$ – tangential displacements on the faces of the k-th layer of the structure;

$W_1^{(k)}(x,y)$, $W_2^{(k)}(x,y)$ – normal displacements on the faces of the k-th layer of the structure,

$W_3^{(k)}(x,y)$, $W_4^{(k)}(x,y)$ – shift functions in the direction of the X and Y axes;

$f_{i1}^{(k)}(z)$, $f_{i2}^{(k)}(z)$, $\beta_1^{(k)}(z)$, $\beta_2^{(k)}(z)$ – predetermined first degree polynomials;

$\varphi_{i1}^{(k)}(z)$, $\varphi_{i2}^{(k)}(z)$, $\beta_3^{(k)}(z)$, $\beta_4^{(k)}(z)$ – second-degree polynomial;

$\varphi_{i3}^{(k)}(z)$, $\varphi_{i4}^{(k)}(z)$ – third-degree polynomials.

The components of the strain tensor of a layer of a structure using the introduced approximation (1) are determined on the basis of the following relations:

$$e_{ii}^{(k)} = U_{il,i}^{(k)} f_{il}^{(k)} + W_{p,ii}^{(k)} \varphi_{ip}^{(k)} + k_{ii} W_p^{(k)} \beta_p^{(k)}; \; e_{33}^{(k)} = W_p^{(k)} \beta_{p,3}^{(k)};$$

$$2e_{12}^{(k)} = U_{1l,2}^{(k)} f_{1l}^{(k)} + U_{2l,1}^{(k)} f_{2l}^{(k)} + W_{p,12}^{(k)} (\varphi_{1p}^{(k)} + \varphi_{2p}^{(k)});$$

$$2e_{i3}^{(k)} = U_{il}^{(k)} f_{il,3}^{(k)} + W_{p,i}^{(k)} (\varphi_{ip,3}^{(k)} + \beta_p^{(k)}). \quad (2)$$

The stress, taking into account the expressions for deformations (2), is written on the basis of Hooke's law:

$$\sigma_{ii}^{(k)} = C_{ii}^{(k)} (U_{il,1}^{(k)} f_{il}^{(k)} + W_{p,ii}^{(k)} \varphi_{ip}^{(k)} + k_{ii} W_p^{(k)} \beta_p^{(k)}) +$$

$$+ C_{ij}^{(k)} (U_{jl,j}^{(k)} f_{jl}^{(k)} + W_{p,jj}^{(k)} \varphi_{1p}^{(k)} + k_{jj} W_p^{(k)} \beta_p^{(k)}) + C_{i3}^{(k)} W_p^{(k)} \beta_{p,3}^{(k)} -$$

$$- (C_{ii}^{(k)} \alpha_i^{(k)} + C_{ij}^{(k)} \alpha_j^{(k)} + C_{i3}^{(k)} \alpha_3^{(k)}) t^{(k)} f_7^{(k)};$$

$$\sigma_{33}^{(k)} = C_{31}^{(k)} (U_{1l,1}^{(k)} f_{1l}^{(k)} + W_{p,11}^{(k)} \varphi_{1p}^{(k)} + k_{11} W_p^{(k)} \beta_p^{(k)}) +$$

$$+ C_{32}^{(k)} (U_{2l,2}^{(k)} f_{2l}^{(k)} + W_{p,22}^{(k)} \varphi_{1p}^{(k)} + k_{22} W_p^{(k)} \beta_p^{(k)}) + C_{33}^{(k)} W_p^{(k)} \beta_{p,3}^{(k)} -$$

$$- (C_{31}^{(k)} \alpha_1^{(k)} + C_{32}^{(k)} \alpha_2^{(k)} + C_{33}^{(k)} \alpha_3^{(k)}) t^{(k)} f_7^{(k)};$$

$$\sigma_{i3}^{(k)} = G_{i3}^{(k)} (U_{il}^{(k)} f_{il,3}^{(k)} + W_{p,i}^{(k)} (\varphi_{ip,3}^{(k)} + \beta_p^{(k)}));$$

$$\sigma_{12}^{(k)} = G_{12}^{(k)} (U_{1l,2}^{(k)} f_{1l}^{(k)} + U_{2l,1}^{(k)} f_{2l}^{(k)} + W_{p,12}^{(k)} (\varphi_{1p}^{(k)} + \varphi_{2p}^{(k)})),$$

$$(i \neq j = 1,2;\; l = 1,2;\; p = 1 \ldots 4). \quad (3)$$

here $f_7^{(k)} = a_0^{(k)} + a_1^{(k)} z + a_1^{(k)} z^2$ (the coefficients $a_0^{(k)}$ and $a_1^{(k)}$ are found on the basis of the solution of the equation of stationary thermal conductivity).

Equations of equilibrium, using expressions for deformations (2) and stresses (3), are obtained on the basis of the Lagrange variational equation.

They have the following form:

$$-B11_{\bar{l}l}^{(k)}U_{1l,11}^{(k)}+TU1_{\bar{l}l}^{(k)}U_{1l}^{(k)}-B611_{\bar{l}l}^{(k)}U_{1l,22}^{(k)}-(B612_{\bar{l}l}^{(k)}+B12_{\bar{l}l}^{(k)})U_{2l,12}^{(k)}-$$
$$-BD11_{\bar{l}p}^{(k)}W_{p,111}^{(k)}-(BD12_{\bar{l}p}^{(k)}+BD611_{\bar{l}p}^{(k)}+BD622_{\bar{l}p}^{(k)})W_{p,221}^{(k)}+$$
$$+(TUW1_{\bar{l}p}^{(k)}+CUW1_{\bar{l}p}^{(k)}-SD1_{\bar{l}p}^{(k)}-k_{11}WC11_{\bar{l}p}^{(k)}-k_{22}WC12_{\bar{l}p}^{(k)})W_{p,1}^{(k)}+$$
$$+BBT1_{\bar{l}}^{(k)}t_{,1}^{(k)}-q_{1\bar{l}}^{(k)}=0;$$
$$-(B612_{\bar{l}l}^{(k)}+B12_{\bar{l}l}^{(k)})U_{1l,12}^{(k)}-B22_{\bar{l}l}^{(k)}U_{2l,22}^{(k)}+TU2_{\bar{l}l}^{(k)}U_{2l}^{(k)}-B622_{\bar{l}l}^{(k)}U_{2l,11}^{(k)}-$$
$$-BD22_{\bar{l}p}^{(k)}W_{p,222}^{(k)}-(BD12_{\bar{l}p}^{(k)}+BD611_{\bar{l}p}^{(k)}+BD622_{\bar{l}p}^{(k)})W_{p,112}^{(k)}+$$
$$+(TUW2_{\bar{l}p}^{(k)}+CUW2_{\bar{l}p}^{(k)}-SD2_{\bar{l}p}^{(k)}-k_{11}WC21_{\bar{l}p}^{(k)}-k_{22}WC22_{\bar{l}p}^{(k)})W_{p,2}^{(k)}+$$
$$+BBT2_{\bar{l}}^{(k)}t_{,2}^{(k)}-q_{2\bar{l}}^{(k)}=0;$$
$$BD11_{\bar{p}l}^{(k)}U_{1l,111}^{(k)}+(BD12_{\bar{p}l}^{(k)}+BD611_{\bar{p}l}^{(k)}+BD622_{\bar{p}l}^{(k)})U_{1l,221}^{(k)}-$$
$$-(TUW1_{\bar{p}l}^{(k)}+CUW1_{\bar{p}l}^{(k)}-SD1_{\bar{p}l}^{(k)}-k_{11}WC11_{\bar{l}p}^{(k)}-k_{22}WC12_{\bar{l}p}^{(k)})U_{1l,1}^{(k)}+$$
$$BD22_{\bar{p}l}^{(k)}U_{2l,222}^{(k)}+(BD12_{\bar{p}l}^{(k)}+BD611_{\bar{p}l}^{(k)}+BD622_{\bar{p}l}^{(k)})U_{2l,112}^{(k)}-$$
$$-(TUW2_{\bar{p}l}^{(k)}+CUW2_{\bar{p}l}^{(k)}-SD2_{\bar{p}l}^{(k)}-k_{11}WC21_{\bar{l}p}^{(k)}-k_{22}WC22_{\bar{l}p}^{(k)})U_{2l,2}^{(k)}+$$
$$+DD11_{\bar{p}p}^{(k)}W_{p,1111}^{(k)}+DD22_{\bar{p}p}^{(k)}W_{p,2222}^{(k)}+$$
$$+(DD12_{\bar{p}p}^{(k)}+DD611_{\bar{p}p}^{(k)}+DD622_{\bar{p}p}^{(k)}+DD612_{\bar{p}p}^{(k)})W_{p,1122}^{(k)}+$$
$$-(TW2_{\bar{p}p}^{(k)}+CC2_{\bar{p}p}^{(k)}+CW2_{\bar{p}p}^{(k)}-ZD2_{\bar{p}p}^{(k)}-k_{11}WO21_{\bar{p}p}^{(k)}-k_{22}WO22_{\bar{p}p}^{(k)})W_{p,22}^{(k)}+$$
$$+(ZZ_{\bar{p}p}^{(k)}+k_{11}W11+k_{22}W12+k_{11}W21+k_{22}W22+$$
$$+k_{11}k_{11}WW11+k_{11}k_{22}WW12+k_{22}k_{11}WW21+k_{22}k_{22}WW22)W_{pc}^{(k)}+$$
$$+DDT1_{\bar{p}}^{(k)}t_{,11}^{(k)}+DDT2_{\bar{p}}^{(k)}t_{,22}^{(k)}+$$

$$+(ZT_{\bar{p}}^{(k)}+k_{11}DT1_{\bar{p}}^{(k)}+k_{22}DT2_{\bar{p}}^{(k)})t^{(k)}-q_{3\bar{p}}^{(k)}=0. \quad (4)$$

For the hinged support of the heat conduction equation has the form:

$$-\frac{\lambda_1^{(k)}}{\lambda_3^{(k)}}(\frac{\pi nx}{a})^2-\frac{\lambda_2^{(k)}}{\lambda_3^{(k)}}(\frac{\pi ny}{b})^2+f_{7,33}^{(k)}=0.$$

It is desirable to realize such an equation by the grid method (it is possible to accurately satisfy the boundary conditions on the surface of the layers, both in temperature and in heat flux, without particular complications in the implementation).

The second derivative with respect to the temperature distribution function over the thickness of the structure is based on the following difference equation:

$$f_{7,33}^{(k)}=\frac{f_{7(i-1)}^{(k)}-2f_{7(i)}^{(k)}+f_{7(i+1)}^{(k)}}{h^{(k)}}.$$

The temperature values determined in this way on the surfaces of the layers and at their center make it easy to determine the coefficients of the polynomial $f_7^{(k)}=a_0^{(k)}+a_1^{(k)}z+a_1^{(k)}z^2$. Further, it is possible to solve the system of equations (4) for hinged support.

2. Results of numerical studies

As an example, the thermo-stressed state of a three-layer hollow shell with the following physical-mechanical characteristics was considered: $E_0=1\,MPa$; $E_1^{(1)}=172\cdot1000\cdot E_0$; $E_2^{(1)}=E_3^{(1)}=6{,}9\cdot1000\cdot E_0$; $G_{12}^{(1)}=G_{13}^{(1)}=3{,}45\cdot1000\cdot E_0$; $G_{23}^{(1)}=1{,}38\cdot1000\cdot E_0$; $\nu_{12}^{(1)}=\nu_{13}^{(1)}=\nu_{32}^{(1)}=0{,}25$ $(\nu_{21}/E_2=\nu_{12}/E_1)$; $\lambda_1^{(1)}=1{,}2\frac{W}{m\cdot deg}$; $\lambda_1^{(1)}=0{,}8\frac{W}{m\cdot deg}$; $\alpha_0=1\,deg$; $\alpha_1^{(1)}=8{,}0\cdot10^{-6}\cdot\alpha_0$; $\alpha_2^{(1)}=\alpha_3^{(1)}=163{,}0\cdot10^{-6}\cdot\alpha_0$; $h^{(1)}=\frac{h}{4}$ $(h=1\,m)$.

The second layer, the $h^{(2)}=\frac{h}{2}$ thickness, identical to the first, but returned to 90^0, the third layer is similar to the first, $\frac{a}{h}=5$ $(a=b)$. The thermomechanical contact of the layers is ideal. A plate loaded on the upper surface by a temperature distributed according to the law of the sine $T_{q31}=50\cdot T_0$ ($T_0=1\,deg$). On the lower surface, the temperature is zero $T_{q32}=0$. Curvature of the shell

$k_{11} = \pm kr1$; $k_{22} = \pm kr2$ $\left(kr1 = \frac{8 \cdot (h/5)}{(a+b)/a};\ kr2 = \frac{8 \cdot (h/5)}{(a+b)/b} \right)$. The calculation was carried out when each layer was considered within the same sublayer (P_1). The accuracy was confirmed by calculation according to the applied model with breakdown of each layer into 128 sublayers (P_128).

The temperature at the boundaries of the layers with consideration of each layer within the same sublayer 50; 34,9685; 10.8110; 0 with breakdown of each layer into 128 sublayers 50; 35,0164; 10.7681; 0.

Table 1 shows dimensionless values of displacements $\bar{U}_1 = \frac{U_1}{h \cdot \alpha_0 \cdot T_0 \cdot 10^{-4}}$, $\bar{U}_3 = \frac{U_3}{h \cdot \alpha_0 \cdot T_0 \cdot 10^{-3}}$ as well as stresses $\bar{\sigma}_{11} = \frac{\sigma_{11}}{\alpha_0 \cdot T_0 \cdot E_0}$ and $\bar{\sigma}_{22} = \frac{\sigma_{22}}{\alpha_0 \cdot T_0 \cdot E_0}$ at the boundaries of layers of a shallow shell with rigid contact between layers and with a rigidly fixed lower surface.

A variant with negative and positive curvature was considered (the Z axis is directed downwards).

Conclusions. The developed approach to the investigation of the thermally stressed state of layered composite shells is constructed and a spatial model is constructed for calculating the thermally stressed state of shallow shells on a rigid basis. The peculiarity of this approach is the assignment of the unknown functions to the outer surfaces of the layers, which makes it possible to break the layer into sublayers, reducing the error of approximation to practically zero. With the use of the model under consideration, the analysis of gently sloping composite shells on a rigid substrate under the effect of a temperature load is performed. The considered example showed that the proposed model provides sufficient accuracy in the calculations of layered shallow shells when considering each layer within a single sublayer. When each layer was broken into 32, 64, 128 sublayers, the result was almost the same.

Table 1. Dimensionless values of displacements and stresses on the boundaries of layers with rigid contact between layers and layers and with a rigidly fixed lower surface

Таблиця 1. Безрозмірні значення переміщень і напружень на кордонах шарів з жорстким контактом шарів і прошарків і з жорстко закріпленою нижньою поверхнею

Layer #	$\bar{U}_1$		$\bar{U}_3$		$\bar{\sigma}_{11}$		$\bar{\sigma}_{22}$	
	P_128	P_1	P_128	P_1	P_128	P_1	P_128	P_1
$k_{11} = -kr1$; $k_{22} = -kr2$								
1	-8.874 -5.052	-8.774 -5.010	-4.654 -2.570	-4.667 -2.581	41.12 11.885	40.08 11.52	-46.38 -34.935	-46.51 -35.06
2	-5.052 -1.818	-5.010 -1.893	-2.570 -.3075	-2.581 -.3070	-36.07 -10.98	-36.08 -11.06	49.34 4.617	45.22 .3799
3	-1.818 0	-1.893 0	-.3075 0	-.3070 0	4.003 .4486	4.677 .4215	-10.97 .3625	-11.19 .3406
$k_{11} = kr1$; $k_{22} = kr2$								
1	-12.12 -7.867	-12.08 -7.868	-4.642 -2.552	-4.659 -2.566	24.67 13.91	24.06 13.89	-47.97 -35.48	-48.07 -35.57
2	-7.867 -2.194	-7.868 -2.266	-2.552 -.3024	-2.566 -.3035	-36.17 -10.96	-36.16 -11.02	36.86 8.464	33.23 4.619
3	-2.194 0	-2.266 0	-.3024 0	-.3035 0	4.701 .4284	5.367 .4182	-10.84 .3462	-11.04 .3379

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ПОБУДОВА ПРОСТОРОВОЇ МОДЕЛІ ДЛЯ РОЗРАХУНКУ ТЕРМОНАПРУЖЕНОГО СТАНУ ПОЛОГИХ ОБОЛОНОК НА ЖОРСТКІЙ ОСНОВІ

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Анотація. Сучасні розрахунки шаруватих плит і оболонок в тривимірній постановці засновані на методиці, де розподіл шуканих функцій по товщині конструкції розшукується методом дискретної ортогоналізації. В даній статті на основі підходів, розроблених авторами, проаналізований термонапружений стан шаруватих композитних пологих оболонок з жорстко закріпленою нижньою поверхнею.

Знаходження розподілу шуканих функцій по товщині конструкції відбувається на основі точного аналітичного рішення системи диференціальних рівнянь. Також розглянутий підхід до дослідження термонапруженого стану шаруватих композитних оболонок та побудовано просторову модель для розрахунку термонапруженого стану пологих оболонок на жорсткій основі. На теперішній час це є вельми актуальна задача при розрахунку дорожніх покриттів мостів. Особливістю такого підходу є віднесення шуканих функцій до зовнішніх поверхонь шарів, що дозволяє розбивати шари на підшари, зменшуючи похибку апроксимації практично до нуля

Для побудови просторової моделі вибирається варіант навантаження температурним навантаженням (за законом синуса) і граничних умов (Нав'є), який призводить до розподілу шуканих функцій в плані плити за тригонометричними гармоніками ряду Фур'є. Залучається поліноміальна апроксимація шуканих функцій по товщині.

З використанням розглянутої моделі проведений аналіз пологих шаруватих композитних оболонок на жорсткій основі під впливом температурного навантаження. Розглянутий приклад засвідчив, що пропонована модель забезпечує достатню точність в розрахунках шаруватих пологих оболонок при розгляданні кожного шару в рамках одного підшару. При розбиванні кожного шару на 32, 64, 128 підшарів отримували практично однаковий результат.

Запропонований підхід може застосовуватися у якості еталонного методу при тестуванні прикладних підходів в розрахунках напружених станів шаруватих пологих композитних оболонок.

Ключові слова: температурні навантаження, дотичні навантаження, термонапружений стан, шаруваті композитні оболонки, напіваналітичний метод кінцевих елементів.

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