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ВИЗНАЧЕННЯ НАДІЙНОСТІ КОНСТРУКЦІЙ З УРАХУВАННЯМ АСИМЕТРІЇ ЗАКОНІВ РОЗПОДІЛУ

DETERMINING STRUCTURAL RELIABILITY CONSIDERING THE ASYMMETRY OF DISTRIBUTION LAWS

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Abstract. In the calculations of structural reliability, it is mainly considered that the laws of distribution of random values of resistance and load effect obey the normal law (Gauss's law).

This law is convenient to use and the most widespread. Therefore, it has found wide application in reliability theory for solving most problems. The law of distribution is symmetric, that is, random variables are distributed symmetrically relative to its center (mathematical expectation). But, as experimental studies show, both the material resistence and the load effect on the structure in most cases are subject to asymmetric laws. The asymmetry of the material resistance can be neglected in most practical cases, but failure to take into account the asymmetry of the load effect can lead to significant errors in determining structural reliability.

The authors chose two laws with different degrees of positive asymmetry to approximate the load distribution, namely gamma and lognormal laws. The normal (symmetric) law was used for the resistence distribution law. The results of reliability calculations that take into account different load distribution laws are presented in the form of a table and a graph. The graph shows the dependence of structural reliability on the reliability index for symmetric (P_N) and asymmetric (P_{NG}, P_{NL}) laws.

All calculations were performed using the Mathcad complex, which allows calculating values with sufficient accuracy. The issue of how to choose the distribution law for the load effect obviously depends on the operation mode of a particular bridge and should be based on appropriate statistical studies.

The purpose of this paper is to show the need for taking into account the law of asymmetry in order to determine the structural reliability. Eurocode norms also require to take into account the asymmetry of distribution laws.

Keywords: structural reliability, normal distribution law, safety factor, asymmetric distribution laws, reliability index.

Introduction. In most calculations of structural reliability, the assumption is made that the laws of the distribution of random variables are subject to a normal law (Gauss's law). This approach, although convenient and widely used, has a significant drawback: it assumes a symmetric distribution relative to the mathematical expectation. However, experimental data confirm that the material resistence and load effects on structures are often subject to asymmetric laws.

Papers[1 - 3] show that the mass of motor vehicles with an extreme load has a multimodal nature of distribution under two modes which corresponds to different types of vehicles.

Paper[4] considered separate problems that required the presentation of joint distributions and reliability indexes of bearing capacity. Certain combinations or differences in resistence and load effect distributions were obtained for this purpose.

In order to take into account the asymmetry of the distribution laws of temporary load effect on bridges, two laws - gamma and lognormal which differ in the degree of asymmetry - are considered in the paper. According to the authors, such laws more adequately correspond to real temporary load effects. In addition, in these distributions, asymmetry depends on the coefficient of variation, which in the future will provide an opportunity to take into account the nature of the load. The performed calculations indicate that ignoring asymmetry can significantly affect the determination of structural reliability and lead to the incorrect assessment of their real operational state. For this, convolution formulas are used, the analytical expressions of which are presented in the paper. Examples of joint distributions based on the proposed formulas are given. They can be asymmetric and significantly differ from the normal distribution when solving problems of structural reliability.

The purpose of this paper is to demonstrate the importance of taking into account the asymmetry of laws when determining structural reliability . This consideration is mandatory according to the Eurocodes [5] and helps more accurately assess the real operating state of structures with different load conditions.

Presenting main material. According to the classical reliability theory [6,7], the reliability of a structure (or its element) is the probability that the value of the generalized safety reserve will have a positive value, i.e.:

$$P = Prob(S > 0)\,, \tag{1}$$

where *P* is structural reliability ; *S* is safety reserve.

The safety reserve is defined as the difference of two random values: the generalized resistance of the element R and the generalized load effect E:

$$S = R - E\,. \tag{2}$$

According to probability theory, when the laws of distribution of *R* and *E* values are known, we can determine the distribution law of safety reserve, which we denote by p_S. Taking into account (1), the reliability of the structure *P* and the value *V=1-P*, which is called the probability of failure, are determined as:

$$V = \int_{-\infty}^{0} p_s(s)ds\,,\ P = \int_{0}^{\infty} p_s(s)ds\,. \tag{3}$$

The geometric meaning of the values *P* and *V* is that their values are equal to the area under the distribution curve of the safety reserve $p_S(s)$. When $s > 0$, this area corresponds to the reliability of the structure *P*. When $s < 0$, the area corresponds to the probability of failure of *V* (Figure 1)

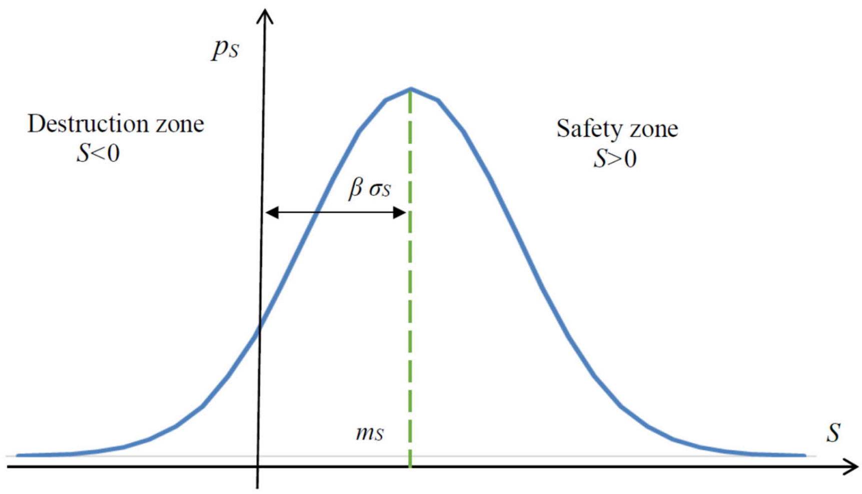

Figure 1 – Distribution function of the safety reserve under the normal distribution law
Рисунок 1 – Функція розподілу резерву міцності при нормальному законі розподілу

In most cases, the values *R* and *E* can be considered independent random variables. Then, under random distribution laws of *R* and *E,* the mathematical expectation of m_s and the mean square deviation of the safety reserve σ_S are determined by the formulas:

$$m_s = m_R - m_E\,,\ \sigma_S = \sqrt{\sigma_R^2 + \sigma_E^2}\ , \tag{4}$$

where m_R , m_E are mathematical expectations, σ_R, σ_E are standards for the distribution of generalized resistance and load effect, respectively.

We denote by *β* the quantity called a reliability index:

$$\beta = \frac{m_S}{\sigma_S}\,. \tag{5}$$

The value of β was first proposed in [6]. It plays an extremely important role in the theory of reliability. As it can be seen from Figure 1, the reliability index determines the number of standards that are placed in the interval from $s = 0$ to $s = m_S$, and this is true for any distribution laws. Taking into account (4), the reliability index β can be written in the form:

$$\beta = \frac{m_R - m_E}{\sqrt{\sigma_R^2 + \sigma_E^2}}. \quad (6)$$

Let us denote by ξ the determined value which is called a reliability index:

$$\xi = \frac{m_R}{m_E}. \quad (7)$$

Then equation (6) takes the form:

$$\beta = \frac{\xi - 1}{\sqrt{C_E^2 + \xi^2 C_R^2}}, \quad (8)$$

where $C_R = \frac{\sigma_R}{m_R}$, $C_E = \frac{\sigma_E}{m_E}$ are coefficients of variation of R and E, respectively.

The formula for determining the reliability index (8) has an advantage over formula (6), because the coefficients of variation can be estimated even with insufficient statistical information regarding the structural resistance and the load effect. In addition, when the load changes or the cross-sectional dimensions of structural elements change, C_R and C_E variations remain unchanged.

In most reliability calculations, distribution laws for the random variables R and E are assumed to be in the form of a normal distribution:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}, \ -\infty < x < \infty. \quad (9)$$

This law is the most studied, so it is widely used in probability theory. The main advantage of this distribution law is stability: the sum or difference of a normal distribution of random variables is also a value under a normal distribution law. Thus, if the random variables R and E have a normal distribution (9), then the safety reserve S will also have a distribution according to the same law, and its parameters m_S and σ_S will be determined according to (4):

$$p_S(s) = \frac{1}{\sqrt{2\pi}\sigma_S} e^{-\frac{(s-m_S)^2}{2\sigma_S{}^2}}. \quad (10)$$

Considering (10), the reliability of P_N structure under normal distribution laws of R and E can be written in the form:

$$P_N = \frac{1}{2} + \Phi(\beta), \quad (11)$$

where $\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{t^2}{2}} dt$ is the Laplace function.

Figure 1 shows a graphical representation of dependence (10). As it can be seen, the normal law is symmetric with respect to the mathematical expectation m, which in this case coincides with the mode M (the value with the greatest probability). In the general case, the distribution law of a random variable can be asymmetric (Figure 2), for which $m \neq M$.

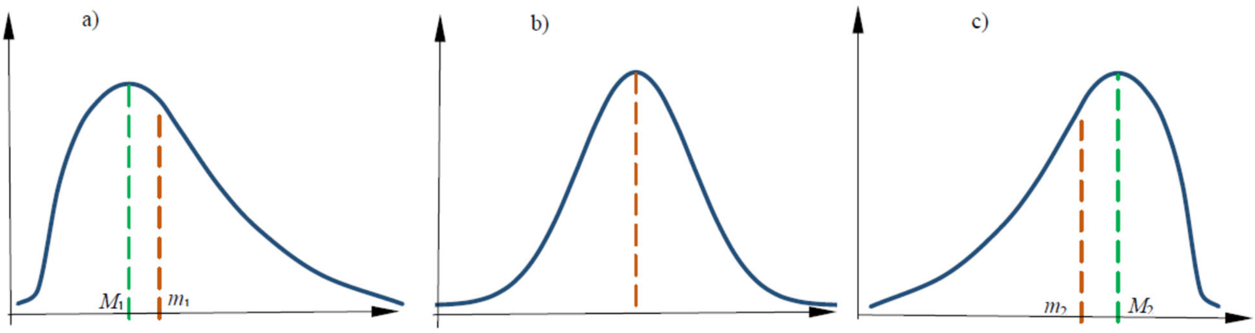

Figure 2 – Graphs of distribution functions
a) positive skewness b) normal distribution law c) negative skewness
Рисунок 2 – Графіки функцій розподілу:
а) з додатньою асиметрією $A > 0$; b) нормальний закон розподілу A=0 c) з відємною асиметрією $A < 0$

The application of formula (11) for calculating reliability is quite simple and convenient, which led to the widespread use of the normal distribution law for resistence and load effect in the form of (9). But this law has two significant shortcomings. The first is that the argument under a normal distribution varies from -∞ to +∞, although in their physical essence both resistence and load effect are purely positive quantities. The second disadvantage is that the normal law is symmetric.

It is known that the asymmetry of the distribution is characterized by the coefficient of skewness A, which in the case of a discrete series of values is equal to:

$$A = \frac{1}{n\sigma^3}\sum_{i=1}^{n}(x_i - m)^3 . \qquad (12)$$

The distribution of a random variable can have both positive (A>0) and negative (A<0) skewness (Figure 2).

Under positive skewness (Figure 2 a), the mathematical expectation m_1 will be greater than the corresponding mode M_1. This means that negative deviations from the average value (for example, reduced load effects) will be more often repeated than positive ones. For the distribution with the coefficient A<0 (Figure 2 c), the situation will be the opposite. That is, load effects that are greater than average will be more often repeated. Asymmetry can be characterized both by the value of A itself and by its relation to the coefficient of a variable A/C.

Experimental studies showed that both the resistence and the load effect are random variables under asymmetric distribution laws. But since asymmetry is a rather insignificant value for resistence (A/C < 1.2), in most cases it can be neglected and it can be assumed that the generalized resistance is distributed according to the normal law, that is, it has a symmetrical shape.

As for the load effect distribution on bridges, based on the analysis of foreign scientific sources, the values of the A/C ratio can reach sufficiently large values (up to 3.5) (unfortunately, we did not find relevant data in domestic scientific sources). As shown by the calculations below, failure to take into account this asymmetry can lead to significant errors in determining structural reliability.

In the paper, the gamma and lognormal distribution laws were chosen to approximate the asymmetric law of load effect distribution with positive skewness. Further in the text, x will denote the value of the random variable *R* (resistance), *y* - the value of the random variable *E* (load effect), and s - the random value of safety reserve *S*.

Gamma distribution:

$$p_{EG}(y) = \frac{\gamma^{\gamma}}{\Gamma(\gamma)m_E}(\frac{y}{m_E})^{\gamma-1}e^{-y\frac{\gamma}{m_E}};\ y > 0, \tag{13}$$

where $\Gamma(\gamma) = \int_0^{\infty} t^{\gamma-1}e^{-t}dt$ is the gamma function, and $\gamma = \frac{1}{C_E^2}$

Lognormal distribution:

$$p_{EL}(y) = \frac{1}{y\sqrt{2\pi\ln(1+C_E^2)}}e^{\frac{-\ln^2(\frac{y}{m_E}\sqrt{1+C_E^2})}{2\ln(1+C_E^2)}};\ y > 0. \tag{14}$$

For the gamma distribution A/C_E =2, and for the lognormal law $A = 3C_E + C_E^3$.

Distributions (13) and (14) are defined for positive values of the argument, therefore, unlike the normal law, they do not contradict the physical essence of the load effect.

For the convenience of calculations and analysis of the obtained results, we introduce dimensionless quantities:

$$\bar{x} = \frac{x}{m_R};\ \bar{y} = \frac{y}{m_R};\ \bar{p} = pm_R. \tag{15}$$

In the dimensionless quantities (15), the distribution laws for load effects (13) and (14) for the generalized resistance (9) will take the form (the sign of the dimensionless quantity is omitted here and further):

$$p_{EG}(y) = \frac{\xi\gamma^{\gamma}}{\Gamma(\gamma)}(\xi y)^{\gamma-1}e^{-\gamma y\xi}, \tag{16}$$

$$p_{EL}(y) = \frac{1}{y\sqrt{2\pi\ln(1+C_E^2)}}e^{\frac{-\ln^2(y\xi\sqrt{1+C_E^2})}{2\ln(1+C_E^2)}}, \tag{17}$$

$$p_{RN}(x) = \frac{1}{\sqrt{2\pi}C_R}e^{-\frac{(x-1)^2}{2C_R^2}}, \tag{18}$$

where *EG*, *EL*, *RN* are indixes denoting the load effect with a gamma distribution, the load effect with a lognormal distribution, and the material resistance with a normal distribution, respectively.

According to the theory of probability, if the random variables *R* and *E* are distributed according to the laws $p_R(x)$ and p_E(y), the law for the distribution of the random variable *S* is determined by the equation:

$$p_s(s) = \frac{d}{ds}\iint_D p_R(x)p_E(y)dxdy. \tag{19}$$

The area of integration D is the intersection of three areas:

- changes in the *x* argument (resistance);
- changes to the *y* argument (load effect):
- regions (x – y) < *s*.

The areas of change in x and y are determined by the accepted distribution laws. In the case of normal laws, the argument will vary from -∞ to +∞, and in the case of asymmetric distribution laws (13) and (14) from 0 to + ∞. When *s* >0, region D is shown in Figure 3.

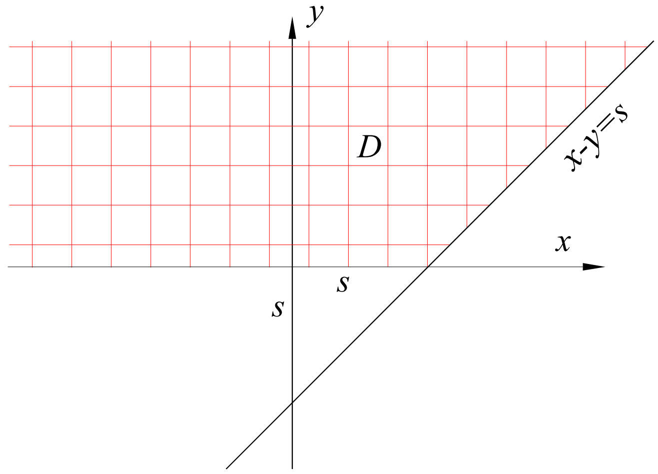

Figure 3 – Integration area D
Рисунок 3 – Область інтегрування D

Using the rule of differentiation of integral functions and taking into account the domain D for a combination of normal and gamma distributions, we obtain the general density of the distribution, which we denote by p_{SNG}:

$$p_{SNG}(s) = \int_0^\infty p_{RN}(y+s)\,p_{EG}(y)\,dy \quad . \tag{20}$$

Similarly, we obtain the distribution density for a combination of normal and lognormal laws:

$$p_{SNL}(s) = \int_0^\infty p_{RN}(y+s)\,p_{EL}(y)\,dy \tag{21}$$

Let us define the reliabilities P_{NG} і P_{NL}, which, according to (3), are the areas under the curves (20) and (21) when s > 0, as

$$P_{NG} = \int_0^\infty p_{SNG}(s)\,ds\,,\;\; P_{NL} = \int_0^\infty p_{SNL}(s)\,ds\,. \tag{22}$$

Table 1 shows the results of calculations of safety and reliability indexes that depend on the safety factor ξ for the selected coefficients of variation of the generalized resistence and load effect C_R = 0,1, C_E=0,35.

ξ	□	P_N	P_{NL}	Δ*100	P_{NG}	Δ*100
1,5	1,3130643	0,90541936	0,90164338	0,420	0,89907291	0,710
1,7	1,7990163	0,96399195	0,94975915	1,500	0,95203785	1,260
2,0	2,4806947	0,99344367	0,98206644	1,160	0,98569175	0,790
2,5	3,4874292	0,99975616	0,99676825	0,300	0,99837171	0,140
3,0	4,3386092	0,99999283	0,99938699	0,060	0,99983221	0,020
3,5	5,0507627	0,99999978	0,99987524	0,010	0,99998319	0,002
4,0	5,6443252	0,99999999	0,99997284	0,003	0,99999829	0,0002

In table 1 Δ denotes the percentage of the relative deviation of the reliability P_{NG} and P_{NL} (22) from the value of P_N, which is calculated according to formula (11).

As it can be seen from the analysis of the results presented in Table 1, for reliability indexes $\beta > 2{,}5$ the deviations of the reliability P_{NG} and P_{NL} from P_N are sufficiently small. This means that the asymmetry of the load effect distribution can be neglected with sufficient accuracy in this case. When β decreases from 2,5 to 1,5 percent, the deviation of the reliability P_{NG} and P_{NL} from P_N increases and can reach $\Delta \approx 1{,}5\%$.

Since the structural reliability, according to the current DSTU [8], can vary in a small range (approximately from 1 to 0,95), the error, which is 1,5%, is quite significant.

Figure 4 presents graphs of P_{NG} , P_{NL} and P_N dependencies on the value of the reliability index β.

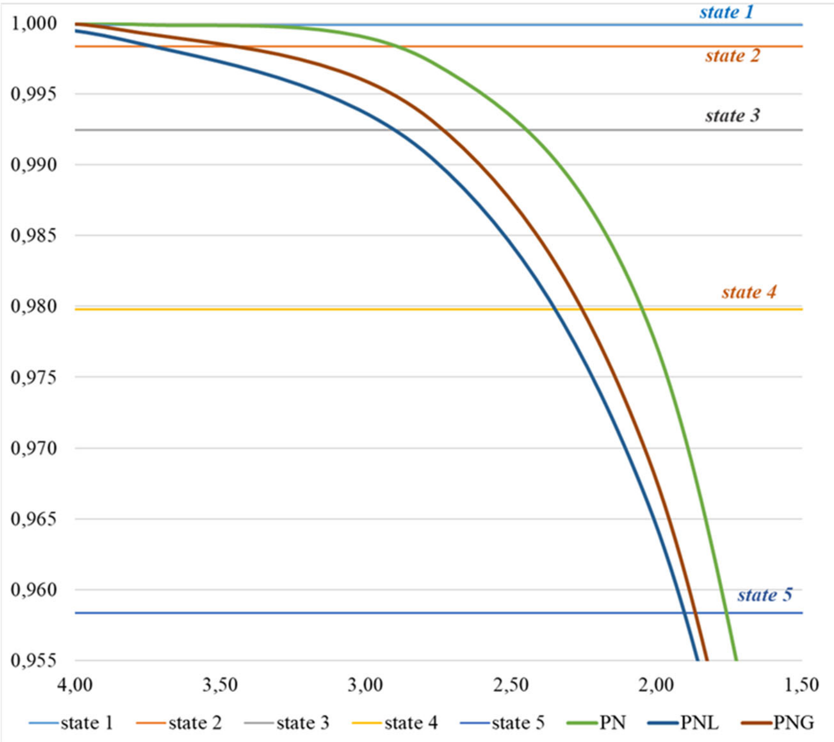

Figure 4 – Dependence of reliability on the reliability index β under symmetric(P_N) and asymmetric(P_{NG}, P_{NL}) distribution laws

Рисунок 4 – Залежність надійності від коефіцієнта безпеки β при симетричному (P_N) і асиметричних (P_{NG} , P_{NL}) законах розподілу

The horizontal lines show the levels of reliability, which according to [8] correspond to the numbers of operational state: from state 1 (in service) to state 5 (out of service). As it can be seen from the above graphs, when $\beta < 2{,}5$, failure to take into account the asymmetry of the distribution law can lead to an overestimated number of the operational state under which the structure actually is . For example, taking into account the normal distribution, if β=2,2, the reliability P_N = 0,9862, which corresponds to operational state 3, but at the same time, taking into account the lognormal distribution, the reliability will be P_{NL} = 0,9724, which corresponds to state 4 of the structure. Such a situation with the determination of states, the numbering of which depends on the level of the bridge maintenance, is quite dangerous. Especially when it concerns structures that are in states 3 or 4.

Summary. The work shows that when calculating the structural reliability in order to determine the structure's operational state, the calculations should be carried out taking into account the asymmetry of the distribution law for the generalized load effect. In most cases, in practice, a symmetrical, normal distribution law can be chosen for the generalized structural resistence. As for asymmetric distribution laws, we propose to choose either the gamma distribution (for the case of medium asymmetry A/C=2) or the lognormal distribution (for large asymmetry A/C≈3). In cases of greater or negative asymmetry, it is necessary to apply more complex (three-parametric) distribution laws, which are not examined in this paper. The definition of the type of asymmetric law should be based on specially conducted experimental studies.

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ВИЗНАЧЕННЯ НАДІЙНОСТІ КОНСТРУКЦІЙ З УРАХУВАННЯМ АСИМЕТРІЇ ЗАКОНІВ РОЗПОДІЛУ

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Висновки. У роботі показано, що при розрахунку надійності конструкції з метою визначення її експлуатаційного стану обчислення слід проводити з урахуванням асиметрії закону розподілу для узагальненого навантаження. В більшості випадків, на практиці, для узагальненої міцності конструкції можна вибирати симетричний, нормальний закон розподілу. В якості асиметричних законів розподілу пропонується вибирати або гамма розподіл (для випадку середньої асиметрії *A/C*=2) або логнормальний розподіл (при великій асиметрії *A/C*≈3). У випадках більшої або від'ємної асиметрії необхідно застосовувати більш складні (трьох параметричні) закони розподілу, які в даній роботі не розглядаються. Визначення типу асиметричного закону має базуватись на спеціально проведених експериментальних дослідженнях.

Ключові слова: надійність конструкцій, нормальний закон розподілу, коефіцієнт безпеки, асиметричні закони розподілу, характеристика безпеки

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