

С помощью предложенной методики можно комплексно решать задачи обеспечения необходимого уровня эффективности перевозок.

Цель исследования - предложить методику моделирования процессов функционирования автотранспортных предприятий и определение оптимальных значений показателей эффективности на основе которых можно получать оптимальные управленческие решения.

Объект исследования - процессы функционирования автотранспортных предприятий.

Поступательное развитие экономики государства ставит все более жесткие условия к качеству, скорости и обеспечения возрастающих объемов перевозок грузов и пассажиров. Это приводит к усилению конкуренции на рынках грузовых и пассажирских перевозок и ставит перед перевозчиками задачи сокращения себестоимости услуг, обновление подвижного состава и других основных фондов. Достичь этого невозможно без применения современных методов прогнозирования, планирования и управления производственными процессами.

Значительное количество перевозчиков имеют до десяти единиц подвижного состава. Даже при наличии желания у таких предприятий в большинстве случаев не хватает ресурсов на инновации и внедрение современных методов управления хозяйственной деятельностью. Управление и планирование даже на большинстве средних и крупных автотранспортных предприятий осуществляется с помощью устаревших и неточных методов, приводит к получению значительных расхождений между желаемыми и достигнутыми результатами.

Учитывая это поиск методов, позволяющих принимать оптимальные решения в процессах управления работой предприятий автомобильного транспорта является важной и актуальной задачей.

**КЛЮЧЕВЫЕ СЛОВА:** ЭКОНОМИКО-МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ, ПРОЦЕСС ФУНКЦИОНИРОВАНИЯ, ОПТИМИЗАЦИЯ, УПРАВЛЕНИЕ, АВТОТРАНСПОРТНЫЕ ПРЕДПРИЯТИЯ.

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#### A MODEL FOR OPTIMIZING SUPPLIES OF PERISHABLE GOODS

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This model addresses the problems of supplies of perishable goods. The obvious example is perishable foodstuffs, for instance, meat, milk, vegetables, bread, but the term can be used in a wider context to include, for example, flowers, newspapers – a daily newspaper has a shelf life of one day. The model also could be used under the circumstances of purchasing by retailers and wholesalers goods which have to be sold during a short-term selling season.

As a rule, a demand for the above-mentioned goods within interval between successive regular deliveries determined by a schedule is not the constant value and it is impossible to forecast this demand absolutely precisely. But managers organizing a sale of these goods have to decide the problem of providing a high level of satisfaction of purchasing demand, and, at the same time, minimizing losses connected with unsold (during periods of their shelf-lives or selling season) goods. This problem could be decided as by using by producers and retailers of technological advances that help to preserve the freshness of perishable goods more long period of time (unfortunately these advances do not guarantee always saving of the same taste of a product and sometimes even they could be injurious to health of consumers), as by rational management by supplies of perishable goods and goods which have to be sold during a short-term selling season.

Well known models of supplies of perishable goods and goods which have to be sold during short-term selling season [1, 133-137] do not analyze a possibility of reaction to random variations of the demand for the goods within the interval between successive regular deliveries by means of transport. The model suggested in the article is proposed to respond to random variations of the demand for the goods between successive regular deliveries by introduction a possibility of an 'emergency' delivery. And the total costs of the suggested model of management by supplies of perishable goods and goods which have to be sold during short-term selling season include the transport costs of the regular consignment, expected transport costs of

the emergency consignment and the expected costs of carrying the unsold goods. The total costs are minimized and the optimal level of the regular consignment is determined.

#### The Analytical Model

Let us assume, initially, that a delivery of a perishable good ( $g$ ) – the supply – is determined by the regular schedule which is connected with the shelf life of that good, for instance a shelf life of one day implies, at least, regular daily deliveries. However, the demand for a good within the interval between successive regular deliveries will not in general be constant value, let us denote this demand by a continuous random variable  $X$  with a probability density function of  $f(x)$ .

It is improbable that the demand in any regular delivery period would exactly match the supply. Thus in the majority of situations there would be two outcomes:

- (i) actual demand would turn out to be less than the supply, in which case the retailer would lose the value of the unsold quantity, and
- (ii) actual demand would turn out to be greater than the supply, in which case the retailer would not satisfy total consumer demand.

A retailer could attempt to meet fully the actual demand, and, at the same time, minimize total cost of supply, by introduction of a possibility of making an emergency order (assuming of course that the supplier and transport operator were agreeable) to cover the expected shortfall. So, if the actual demand ( $X$ ) were greater than the regularly scheduled supply ( $g$ ) the retailer would order an emergency delivery of size ( $X - g$ ); if the actual demand were less than the regularly scheduled delivery, then the retailer would carry the loss of the unsold good of size ( $g - X$ ).

The total costs,  $S(g)$ , of the supply of the good in the scheduled period may therefore be expressed as the sum of the transport costs of the regular consignment  $S_1(g)$ ; the expected transport costs of the emergency consignment  $S_2(g)$ ; and the expected cost of carrying of the unsold good,  $S_3(g)$ :

$$S(g) = S_1(g) + S_2(g) + S_3(g) \quad (1)$$

If we take into account that the costs of transport of a good on a delivery route is are determined by such linear expression as [2, p. 107-108] (Appendix 1)

$$ag + b, \quad (2)$$

and also that the probability distribution of the demand when an emergency delivery has been ordered is characterized by a truncation of the upper-tail, and can be expressed by

$$\varphi(x) = \frac{f(x)}{1 - \int_0^g f(x)dx}, \quad \text{where } x \geq g, \quad (3)$$

then the components of equation (1) may be expressed as

$$S_1(g) = a_1g + b_1, \quad (4)$$

$$S_2(g) = a_2 \int_g^\infty (x - g)f(x)dx + b_2 \left( 1 - \int_0^g f(x)dx \right), \quad (5)$$

$$S_3(g) = \Delta C \int_0^g (g - x)f(x)dx, \quad (6)$$

where  $a_1$ ,  $b_1$  and  $a_2$ ,  $b_2$  – coefficients of linear expressions that determine the transport costs for the regular and emergency consignments accordingly;

$\Delta C$  – the value of the loss of one unsold item of the good.

To determine the optimal level of the regular (that is the initial) consignment, it is necessary to partially differentiate the expression for  $S(g)$  with respect to  $g$ , and then set  $\partial S(g) / \partial g = 0$ .

Carrying out this partial differentiation yields:

$$a_2 - a_1 + b_2 f(g_{opt}) - (a_2 + \Delta C) \int_0^{g_{opt}} f(x) dx = 0 \quad (7)$$

And then the decision of the task lies in determining of the probability distribution of the demand in the regular delivery period  $f(x)$  and solving equation (7) with regard to  $g_{opt}$  – the optimal level of the regular consignment.

The demand for perishable goods is influenced to many factors and we could expect that it is described by the normal probability distribution. So, equation (7) could be expressed as

$$a_2 - a_1 + \frac{b_2}{\sigma_x \sqrt{2\pi}} e^{-\frac{(g_{opt}-m_x)^2}{2\sigma_x^2}} - \frac{a_2 + \Delta C}{\sigma_x \sqrt{2\pi}} \int_{-\infty}^{g_{opt}} e^{-\frac{(g_{opt}-m_x)^2}{2\sigma_x^2}} g_{opt} dx = 0, \quad (8)$$

where  $m_x$  – expected value of the demand in the regular delivery period;

$\sigma_x$  – standard deviation of the demand in the regular delivery period.

Let us take 
$$\frac{g_{opt} - m_x}{\sigma_x} = t_{opt} \quad (9)$$

and put it in the equation (8). Then we receive

$$a_2 - a_1 + \frac{b_2}{\sigma_x \sqrt{2\pi}} e^{-\frac{t_{opt}^2}{2}} - \frac{a_2 + \Delta C}{\sigma_x t_{opt}} \Phi^*(t_{opt}) = 0, \quad (10)$$

where

$$\Phi^*(t_{opt}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t_{opt}} e^{-\frac{z^2}{2}} dz.$$

The decision of the equation (10) with respect to  $t_{opt}$  determines the optimal level of the regular consignment:

$$g_{opt} = m_x + \sigma_x \cdot t_{opt}. \quad (11)$$

Equation (10) has no accurate decision and can be solved by using of approximation.

It makes sense also to analyse the function of total costs (1), (appendix 2):

$$\begin{aligned} S(g) = & a_1 g + b_1 + a_2 \left[ m_x - g + (g - m_x) \Phi^* \left( \frac{g - m_x}{\sigma_x} \right) + \sigma_x \varphi \left( \frac{g - m_x}{\sigma_x} \right) \right] + \\ & + b_2 \left[ 1 - \Phi^* \left( \frac{g - m_x}{\sigma_x} \right) \right] + \Delta C \left[ (g - m_x) \Phi^* \left( \frac{g - m_x}{\sigma_x} \right) + \sigma_x \varphi \left( \frac{g - m_x}{\sigma_x} \right) \right]. \end{aligned} \quad (12)$$

The character of the function of the total costs  $S(g)$  essentially depends of the value of the loss of one unsold item of the good, parameters of the distribution of the demand for the good and factors determining the costs of transportation, figure 1.

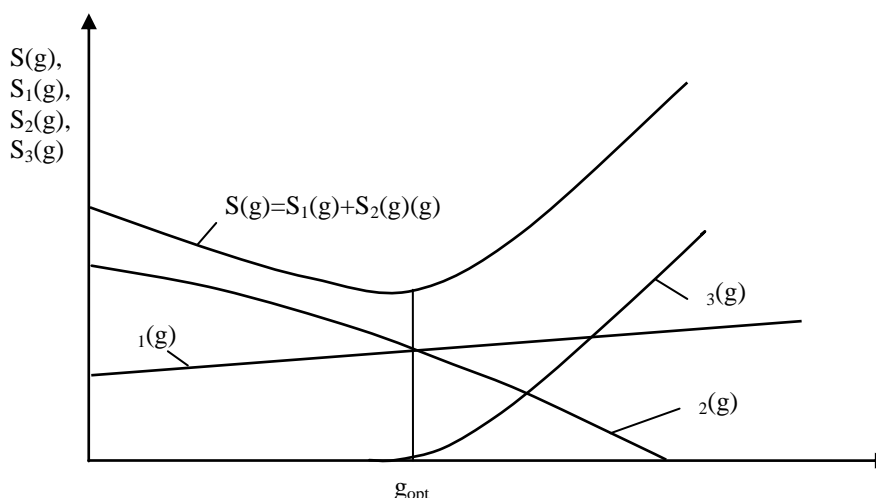


Figure 1. – The function of the total costs and its components  
Implementation Experience

In Ukraine first time the suggested model was applied under the circumstances of supplies of the bread.

In Tsarist Russia (part of which Ukraine was), bread was traditionally produced and sold mostly by local bakers – often individuals or family concerns. With the advent of the Soviet Union, local bakeries were abolished and in their place large-scale bakeries were set up. The bread was then sold in shops, department stores and supermarkets but consumer taste, and demand, for fresh bread persisted, and customers aspired to buy freshly-baked bread if it were available. In that case the producer and the consumer were far apart.

Since the demise of the Soviet Union in 1991 and establishment of the independence of Ukraine, the trend has been from the planned to the market-oriented economy; small-scale bakers have re-appeared but the greater part (70%) of the supplies of the bread today is still met by large-scale enterprises. There is still, of course, the demand for fresh bread, and in this context the consumer is benefiting, not only by the reappearance and proximity of the local baker, and by technological advances that help to preserve the freshness of the product more long period of time, but also by improving efficiency in the logistic chain "bread-produced enterprise - transport operator – retailer".

The introduction of the model provides:

- high level of satisfaction of the purchasing demand for the fresh bread;
- cutting of the total costs in the logistic chain "bread-produced enterprise - transport operator retailer" to 9%.

The transport operators, providing appropriate service of the retailers started to apply the emergency delivery in accordance with the suggested model, have to create the reserve of trucks. A model of optimizing the quantity of trucks in the reserve was also suggested by the author. It could be demonstrated in the future publications.

#### Summary

The model for optimizing supplies of perishable goods has been described in the article. This model is proposed to react to random variations of the demand for the perishable goods between successive regular deliveries by introduction of a possibility of the emergency delivery. And the total costs of the suggested model include the transport costs of the regular consignment, expected transport costs of the emergency consignment and the expected costs of carrying the unsold goods. The total costs are minimized and the optimal level of the regular consignment is determined. The model also could be used under the circumstances of supplies of goods which have to be sold during a short-term selling season.

A model of optimizing the quantity of trucks for the purposes of making the emergency delivery is also suggested. It could be demonstrated in the future publications.

## APPENDIX 1

$$a = \frac{1}{q\gamma} \left[ C_{km}(2\bar{l}_i - \bar{l}_{(i-1)-i}) + C_{fix}(t_l - t_a) \right], \quad (1)$$

$$b = C_{km} \bar{l}_{(i-1)-i} + C_{fix} t_a, \quad (2)$$

where  $q$  – nominal capacity of a truck,  $t$ ;  $\gamma$  – load factor;  $\bar{l}_i$  – average radial distance, km;  
 $C_{km} = C_{var} - \frac{C_{fix}}{V_0}$  – costs per 1 km of ride of a truck, grn.;  $V_0$  – operational speed, km/h;  $l_{(i-1)-i}$  – average leg between successive points of the destination on a delivering route, km;  $t_l$  – time of load and unload, h;  
 $C_{km}$  – variable costs per 1 km of ride of a truck, grn.;  $C_{fix}$  – fixed costs on an hour of truck's work, grn.;  $t_a$  – additional time for visiting to each point of the destination on a delivery route, h.

## APPENDIX 2

During determination of integrals  $\int_{-\infty}^g f(x)xdx$  and  $\int_g^{\infty} f(x)xdx$  we used formula for a probability density function of normal distribution which truncation of the upper-tail ( $x \geq g$ )

$$f_{tr}(x) = \frac{1}{\sigma_x(1-\tau)} \varphi\left(\frac{x-m_x}{\sigma_x}\right) \quad (1)$$

where  $\varphi\left(\frac{x-m_x}{\sigma_x}\right)$  – probability density function of normal distribution  $\tau = \Phi^*\left(\frac{g-m_x}{\sigma_x}\right)$  – level of truncation of the upper-tail and formula for expected value of truncation random variable

$$m_{tr} = m_x + \sigma_x^2 f_{tr}(g) \quad (2)$$

As

$$\int_g^{\infty} f_{tr}(x)xdx = m_x + \frac{\sigma_x}{1-\Phi^*\left(\frac{g-m_x}{\sigma_x}\right)} \varphi\left(\frac{g-m_x}{\sigma_x}\right)$$

and for  $X \geq g$

$$f_{tr}(x) = \frac{1}{1-\Phi^*\left(\frac{g-m_x}{\sigma_x}\right)} f(x),$$

so

$$\int_g^{\infty} f_{tr}(x)xdx = m_x \left[ 1 - \Phi^*\left(\frac{g-m_x}{\sigma_x}\right) \right] + \sigma_x \varphi\left(\frac{g-m_x}{\sigma_x}\right)$$

If we take into account that

$$\int_{-\infty}^g f(x)xdx = m_x - \int_g^{\infty} f(x)xdx$$

so

$$\int_{-\infty}^{\infty} f(x)xdx = m_x \Phi^* \left( \frac{g - m_x}{\sigma_x} \right) - \sigma_x \varphi \left( \frac{g - m_x}{\sigma_x} \right)$$

After transformation the function of the total costs can be presented as:

$$S(g) = a_1 g + b_1 + a_2 \left[ m_x - g + (g - m_x) \Phi^* \left( \frac{g - m_x}{\sigma_x} \right) - \sigma_x \varphi \left( \frac{g - m_x}{\sigma_x} \right) \right] + \\ + b_2 \left[ 1 - \Phi^* \left( \frac{g - m_x}{\sigma_x} \right) \right] + \Delta C \left[ (g - m_x) \Phi^* \left( \frac{g - m_x}{\sigma_x} \right) + \sigma_x \varphi \left( \frac{g - m_x}{\sigma_x} \right) \right]$$

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2. Воркут А.И. Грузовые автомобильные перевозки. – К.: Вища школа, 1986. – 447 с.

#### РЕФЕРАТ

Воркут Т.А., Корнієнко О.В. Модель оптимізації постачань швидкопсувних товарів. / Тетяна Анатоліївна Воркут, Євгеній Леонідович Приліпко, Олена Валеріївна Корнієнко // Управління проектами, системний аналіз і логістика. – К.:НТУ – 2012. – Вип. 10.

Проведений, з використанням розробленої класифікації моделей управління запасами швидкопсувних продуктів повсякденного попиту, аналіз показує, що найбільш ефективна двоступенева організація поставок, яка передбачає можливість додаткового завозу спеціально виділеними транспортними засобами. Аналіз сумарних витрат, які визначаються з урахуванням ймовірностей вимог на додаткове завезення у випадку дефіциту, і втрат через утворення залишків при зайвих поставках, показує, що вибір величини початкової поставки є оптимізаційною задачею. Одержано залежність, що встановлює вплив на оптимальну величину початкової поставки параметрів виявленого нормального розподілу попиту за інтервал часу між черговими поставками, експлуатаційних факторів, що визначають показники використання автомобілів на розвізних маршрутах, і величини втрат на одиницю нереалізованої продукції.

**КЛЮЧОВІ СЛОВА:** ПОСТАЧАННЯ, ШВИДКОПСУВНІ ПРОДУКТИ, МОДЕЛЬ ОПТИМІЗАЦІЇ ПОСТАЧАНЬ.

#### ABSTRACT

T.A. Vorkut, Kornienko O.V. A model for optimizing supplies of perishable goods. / Tatiana Vorkut, Eugene Prylipko, Evgen Prilipko, Olena Kornienko // Management of projects, system analysis and logistics. – К.: NTU – 2012. – Vol. 10.

This model addresses the problems of supplies of perishable goods. The obvious example is perishable foodstuffs, for instance, meat, milk, vegetables, bread, but the term can be used in a wider context to include, for example, flowers, newspapers – a daily newspaper has a shelf life of one day. The model also could be used under the circumstances of purchasing by retailers and wholesalers goods which have to be sold during a short-term selling season.

As a rule, a demand for the above-mentioned goods within interval between successive regular deliveries determined by a schedule is not the constant value and it is impossible to forecast this demand absolutely precisely. But managers organizing a sale of these goods have to decide the problem of providing a high level of satisfaction of purchasing demand, and, at the same time, minimizing losses connected with

unsold (during periods of their shelf-lives or selling season) goods. This problem could be decided as by using by producers and retailers of technological advances that help to preserve the freshness of perishable goods more long period of time (unfortunately these advances do not guarantee always saving of the same taste of a product and sometimes even they could be injurious to health of consumers), as by rational management by supplies of perishable goods and goods which have to be sold during a short-term selling season.

Well known models of supplies of perishable goods and goods which have to be sold during short-term selling season do not analyze a possibility of reaction to random variations of the demand for the goods within the interval between successive regular deliveries by means of transport. The model suggested in the article is proposed to respond to random variations of the demand for the goods between successive regular deliveries by introduction a possibility of an 'emergency' delivery. And the total costs of the suggested model of management by supplies of perishable goods and goods which have to be sold during short-term selling season include the transport costs of the regular consignment, expected transport costs of the emergency consignment and the expected costs of carrying the unsold goods. The total costs are minimized and the optimal level of the regular consignment is determined. The model also could be used under the circumstances of supplies of goods which have to be sold during a short-term selling season.

A model of optimizing the quantity of trucks for the purposes of making the emergency delivery is also suggested. It could be demonstrated in the future publications.

**KEYWORDS: SUPPLIES, PERISHABLE GOODS, MODEL FOR OPTIMIZING SUPPLIES.**

#### РЕФЕРАТ

Воркут Т.А., Прилипко Е.Л., Корниенко Е.В. Модель оптимизации поставок скоропортящихся товаров. / Татьяна Анатолиевна Воркут, Евгений Леонидович Прилипко, Елена Валериевна Корниенко // Управления проектами, системный анализ и логистика. – К.:НТУ – 2012. – Вып. 10.

Проведенный, с использованием разработанной классификации моделей управления запасами скоропортящихся продуктов повседневного спроса, анализ показывает, что наиболее эффективна двухступенчатая организация поставок, которая предусматривает возможность дополнительного завоза специально выделяемыми транспортными средствами. Анализ суммарных затрат, определяемых с учетом вероятностей требований на дополнительный завоз в случае дефицита, и потерь из-за образования остатков при излишних поставках, показывает, что выбор величины первоначальной поставки является оптимизационной задачей. Получена зависимость, устанавливающая влияние на оптимальную величину первоначальной поставки параметров выявленного нормального распределения спроса за интервал времени между очередными поставками, эксплуатационных факторов, определяющих показатели использования автомобилей на развозочных маршрутах, и величины потерь на единицу нереализованной продукции.

**КЛЮЧЕВЫЕ СЛОВА: ПОСТАВКИ, СКОРОПОРТЯЩИЕСЯ ПРОДУКТЫ, МОДЕЛЬ ОПТИМИЗАЦИИ ПОСТАВОК.**