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ЕКРАНУВАННЯ РОЗРИВНИХ ХВИЛЬ ШАРУВАТИМ ПРУЖНИМ АНІЗОТРОПНИМ СЕРЕДОВИЩЕМ

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SHIELDING DISCONTINUOUS WAVES BY ANISOTROPIC ELASTIC LAYERED MEDIA

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ЭКРАНИРОВАНИЕ РАЗРЫВНЫХ ВОЛН СЛОИСТОЙ УПРУГОЙ АНИЗОТРОПНОЙ СРЕДОЙ

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Statement of the problem.

There is a large variety of problems of propagation of waves generated by impact action and characterized by discontinuities of strains and stresses at their front surfaces. Every so often in this cases, it is not necessary to study the field functions behaviour in the whole domain of a medium, but is expedient to consider the process of the wave motion as a signal, which can identify the mechanical system properties. This kind of problems is associated with the reconnaissance of mineral resources via the use of explosions, with the calculation of the phenomena of reflection – refraction of seismic waves in tectonic structures, as well as with the questions of the wave propagation and decay in layered composite materials. Their solutions can help us to attenuate actions of there waves through the use of shielding effects.

In these cases, the problem of investigation of discontinuous waves propagation in elastic media is connected with the questions of geometrical construction of moving field functions discontinuities and calculation of their polarizations and magnitudes presenting the most comprehensive information on a wave front and intensity of the impulse carried by it at the every point of the front.

Analysis of Publications. The methods of geometrical optics, specifically, a zero approach of the ray method, providing an adequate quantitative description of a wide class of the wave phenomena of different physical nature, play an important role in the setting up and solution of the mentioned problems (Karal and Keller [1]; Ogilvy [2]; Petrashen [3]; Podilchuk and Rubtsov [4]).

The ray method allows to separate a function of the wave optical way length (or eikonal) and to construct a system of rays and fronts of a discontinuous wave with the help of the eikonal equation. This problem may be solved comparatively easy for isotropic media, but there again some difficulties arise when there is a necessity to investigate the interaction of a wave with the interface between the media with differing optical properties (lenses, inhomogeneities, etc.). In these events, the aggregates of the rays are produced, which have common envelopes (caustics), where the rays are focussed and the field intensity increases indefinitely. In geometrical optics, the caustics classification is performed on the basis of the theory of singularities of differentiable mappings – the theory of catastrophes (Arnold [5], Kravtsov and Orlov [6]).

But the physical pictures of the dynamic phenomena are drastically complicated when discontinuous waves propagation in anisotropic elastic media is investigated, because of the field functions become vectorial; there are three kinds of waves for every ray direction, which are distinguished by their polarization; the waves phase velocities depend on both the waves polarization and direction of propagation; the ray velocities differ from the phase ones and there is not always one-to-one conformity between their directions. The phenomenon of the wave diffraction in the interfaces between elastic anisotropical media also is radically complicated, as the appropriate correlations by Snellius become essentially non-linear, because of the fact that the phase velocities of the reflected and refracted waves propagation cease to be known beforehand. For this reason it is necessary to solve systems of non-linear equations in order to determine the directions of the rays emanated from the boundary surfaces. The possible non-uniqueness of their solutions may cause the advent of caustics even at the incidence of a regular shock wave on a plane interface, which cannot be in homogeneous isotropical media, and generate more wide diversity of qualitatively different phenomena in the processes of reflection-refraction.

The problems of interaction of incident waves with the boundary surfaces interfacing anisotropical elastic media are normally solved through the construction of the refraction vector functions (Fedorov [7]), which, in essence, represents a graphical method. (Ogilvy [2]). applied a similar approach to investigation of a mirage phenomena in anisotropic heterogeneous elastic media. In the present paper, the method of continuation by a parameter jointly with the Newton method (Gulyayev et al. [8, 9]) is used, which permits to identify bifurcational states of wave front transformation with the best efficiency.

Constitutive equation.

Let the motion of homogeneous anisotropical elastic medium characterized by the elasticity parameters $C_{ki,pq} = const$ and density $\rho = const$ be described by the equations

$$\sum_{k,p,q=1}^{3} \lambda_{ki,pq} \frac{\partial^2 u_q}{\partial x_k \partial x_p} - \frac{\partial^2 u_i}{\partial t^2} = 0 \qquad (i=1,2,3) \tag{1}$$

where $\lambda_{ki,pq} = C_{ki,pq} / \rho$; x_1, x_2, x_3 is the Cartesian coordinate system; u_1, u_2, u_3 the components of the elastic displacement vector.

Consider the system (1) solutions in the form of a plane monochromatic wave with the wave number *k* and the phase velocity *v*. Its front are the surfaces possessing the constant phases

$$\vec{n} \cdot \vec{r} - vt = const, \tag{2}$$

which coincide locally with the areas perpendicular to the unit vector $\vec{n}$ and moving with the velocity $\vec{v} = v\vec{n}$.

The question of the wave polarization vector $\vec{A}$ and phase velocity $\vec{v}$ determination for the selected direction $\vec{n}$ is solved on the basis of the system of homogeneous algebraic equations relative A_i
(Fedorov, [7]).; Gulyayev et al. [8, 9], 1997; Petrashen [3])

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq}\, n_k\, n_p\, A_q - v^2 A_i = 0 \qquad (i = 1,2,3) \tag{3}$$

Its matrix

$$\Lambda_{ik} = \sum_{k,p=1}^{3} \lambda_{ik,pq}\, n_k\, n_p \qquad (i,\, q = 1,2,3). \tag{4}$$

possesses the properties of symmetry and positive definiteness.

From condition of the system (3) non-trivial solution existence, the eigen-value problem stems

$$\left| \sum_{k,p=1}^{3} \lambda_{ik,pq}\, n_k\, n_p - v^2 \delta_{iq} \right| = 0 \tag{5}$$

It is possible to find three values of the velocity

$$v_1^2(\vec{n}) > v_2^2(\vec{n}) \geq v_3^2(\vec{n}) > 0 \tag{6}$$

with the help of Eq. (5) and three types $A^{(r)}$ (*r = 1,2,3*) of the wave polarization

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq}\, n_k\, n_p\, A_q^{(r)} - v_r^2 A_i^{(r)} = 0 \qquad (i = 1,2,3), \tag{7}$$

for each direction $\vec{n}$. The waves are quasi-Primary (qP) and quasi-Secondary (qS).

These polarization vectors satisfy the orthogonality condition

$$\vec{A}^{(i)}(\vec{n}) \cdot \vec{A}^{(k)}(\vec{n}) \qquad (\mathrm{i} = 1,2,3) \tag{8}$$

for every selected direction $\vec{n}$.

If a discontinuous wave is considered, equality (2) provides, that its front surface may be represented by the correlation

$$\tau(x_1, x_2, x_3) - t = 0\,, \tag{9}$$

where the function $\tau(x_1, x_2, x_3)$ has to satisfy the first order partial differential equation (Fedorov [7], Petrashen [3])

$$\sum_{i,k,p,q=1}^{3} \lambda_{ik,pq}\, p_k\, p_p\, A_q^{(r)} A_i^{(r)} = 1\,. \tag{10}$$

It generalizes the eikonal equation of geometrical optics to the case of anisotropic elastic waves.

The quantities p_k (k = 1, 2, 3) included into (10) represent the components of the refraction vector $p_k \equiv \partial\tau / \partial x_k = n_k / v_r(\vec{n})$ (k = 1,2,3).

For the wave front (9) to be constructed, Eq. (10) solution should be found. The equation is transformed to the system of ordinary differential equations

$$\frac{\partial x_k}{\partial \tau} = \sum_{i,k,p,q=1}^{3} \lambda_{ik,pq}\, n_k\, p_p\, A_q^{(r)} A_i^{(r)}\,, \qquad \frac{\partial p_k}{\partial \tau} = 0 \qquad (k = 1,2,3) \tag{11}$$

with the help of the method of characteristics.

The first group of these equations describes the wave propagation along rays with the ray velocity $\xi = \xi^{(r)}(\vec{n}, x_k)$. The rays are rectilinear and, in the general case, not orthogonal to the wave front surface.
The built up system of rays and fronts allows to proceed to the determination of the wave intensity in the vicinity of its front. For the realization to be performed, it is convenient to use Eq. (1) solution expansion in series along a ray as follows:

$$u_q = \sum_{m=0}^{\infty} u_q^{(m)}(x_1, x_2, x_3) f_m\left[t - \tau(x_1, x_2, x_3)\right] \qquad (q = 1,2,3), \tag{12}$$

where the functions f_m, satisfying the correlations $f'_m(y) = f_{m-1}(y)$, are supposed to possess discontinuities of their derivatives, for example, of the order n+2 (Petrashen [3]).
If the problem of investigation of the wave behaviour in the front nearest neighbourhood is set up, only one term m = 0 is retained in (12) and for the vector $\vec{u}^{(0)}$ to be calculated, the system of homogeneous equations

$$\sum_{k,p,q=1}^{3} \lambda_{ik,pq}\, p_k\, p_p\, u_q^{(0)} - u_i^{(0)} \qquad (i = 1,2,3) \tag{13}$$

is used. Its solution is represented in the form (Petrashen [3])

$$u_q^0 = \frac{c_0(\alpha, \beta) \cdot A_q^{(r)}(\alpha, \beta, \tau)}{\sqrt{J(\alpha, \beta, \tau)}} \qquad (q{=}1,2,3) \tag{14}$$

where (α,β,τ) is the system of ray coordinates and the functional determinant $J=\partial(x_1,x_2,x_3)/\partial(\alpha,\beta,\tau)$ of the transformation of the ray coordinates into Cartesian ones is the measure of the ray divergence in the ray tube.

The presented correlations permit to trance the evolution of a discontinuous wave front and to calculate magnitudes of the field functions discontinuities on its surface outside the interface between anisotropic elastic media with differing properties.

Discussion of results.

At first consider the peculiarities of a discontinuous wave propagation in an unbounded transversally isotropic medium. Let a normal uniform pressure is instantly applied to the surface of a spherical cavity *C* of radius *R* = 1. It initiates not only a qP discontinuous wave, as it occurs in an isotropic medium, but also qS discontinuous waves whose front surfaces possess axial symmetry. Thanks to it intensity of the qS-wave (*r* = 3) polarized orthogonally to the first ones (*r* = 1, 2) is equal to zero and hereafter it will not be investigated. The problem is to construct the evolving surfaces of these waves fronts and to investigate their bifurcations.

At the case under study the tensor of elastic constants is represented by the matrix

$$\left(C_{\alpha\beta}\right)=\begin{Vmatrix} L & 0 \\ 0 & M \end{Vmatrix}; \qquad L=\begin{Vmatrix} \lambda+2\mu & \lambda-l & \lambda \\ \lambda-l & \lambda+2\mu-p & \lambda-l \\ \lambda & \lambda-l & \lambda+2\mu \end{Vmatrix},\ M=\mathrm{diag}(\mu-m,\mu,\mu-m).$$

Thus, the three parameters *l, m, p* characterize the considered media difference from an isotropic medium with the Lame parameters λ,μ.

With the use of the proposed approach, the problem on diffraction of a plane discontinuous wave by plane interfaces between two elastic semispaces was studied. The shielding properties of the interfaces were analyzed. Two types of the elastic media joined in the interfaces were simulated.

Firstly, it was considered that conditions of rigid connection are satisfied, i. e. the vectors of displacements and stresses were assumed to coincide. Shown in Fig. 1 are the diagrams of the values of the reflected and refracted waves intensities. It can be concluded that the diffractional character of the waves transformation becomes more intensive with change of the incident angle.

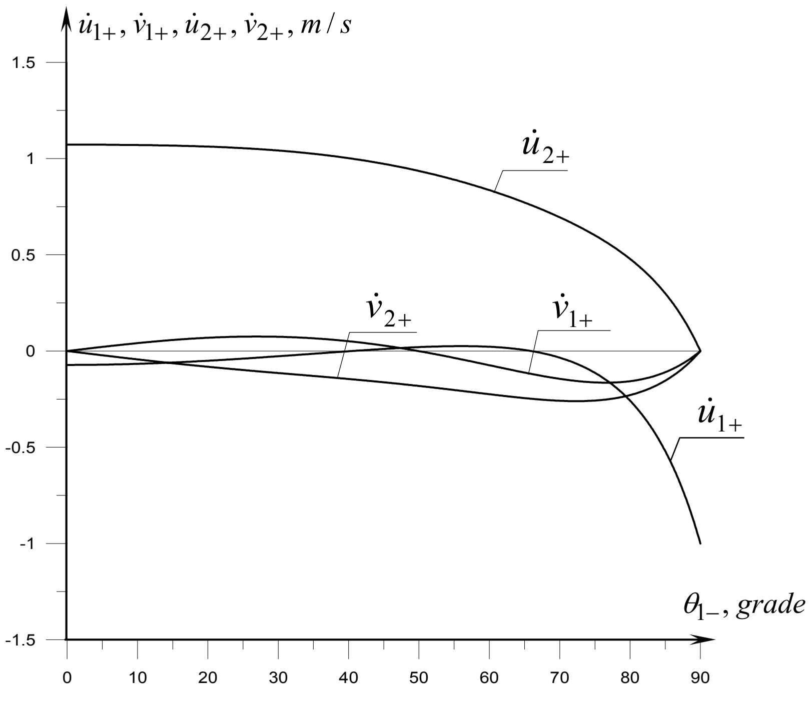

Figure 1 – Intensities of discontinuous waves reflected and refracted in the plane G_1 joining two elastic media

But the shielding phenomena become more intensive when the contacting media can slide in the interface plane. It can be seen in Fig. 2, that the intensities of refracted waves tend to zero with the incident angle enlargement.

The reflection – refraction processes acquire special complexity if the shielding layer has the shape of a wedge (Fig. 3). In this case the screening effect is effective for any incident angle.

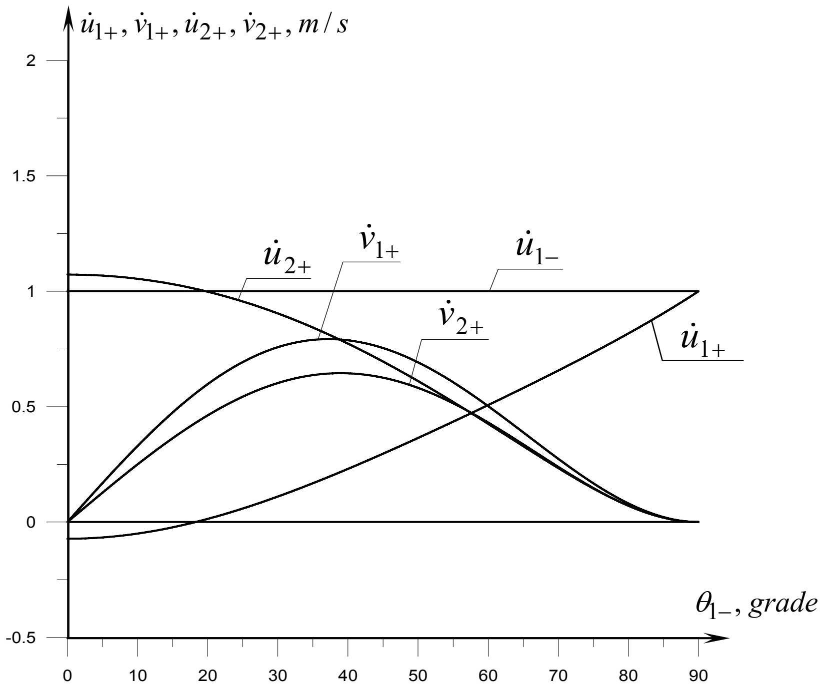

Figure 2 – Intensities of discontinuous waves reflected and refracted in the plane G_1 separating two elastic media

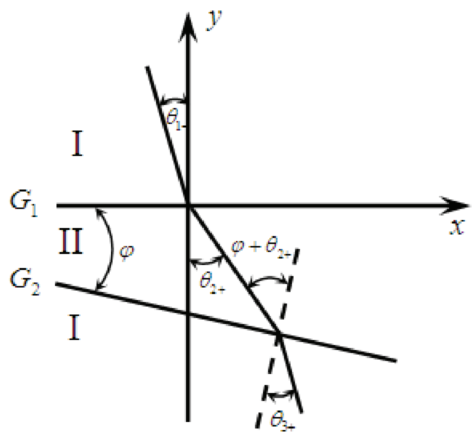

Figure 3 – Scheme of the wave breaking by a wedge insert

Conclusions.

The problem of diffraction of discontinuous waves fronts at the interfaces between anisotropic elastic media is set up in the framework of a zero approximation of the ray theory. Software is elaborated with its use for the computer simulation of the waves fronts transformation and analysis of their field functions discontinuities magnitudes evolution.

The carried out calculations allowed to investigate phenomena of bifurcations of the wave front surfaces and trace the generation of reflected and refracted discontinuous waves in the interface between the media with different mechanical properties. It is established that three factors may be the reasons of the fronts bifurcations and caustics formation. Among these are the non-linear correlation between the directions of rays and a wave normal, the non-linear character of the Snellius equations and non-linear outline of interfaces. In the vicinity of the wave front bifurcations, the field functions intensity tends to infinity.

The effects of the discontinuous wave shielding are established.

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РЕФЕРАТ

Ващіліна О.В. Екранування розривних хвиль шаруватим пружним анізотропним середовищем. / О.В. Ващіліна, Ю.О. Заєць // Вісник Національного транспортного університету. – К. : НТУ, 2013. – Вип. 28.

Розглянута проблема динамічної взаємодії хвильових фазових фронтів з поверхнями розділу анізотропних пружних середовищ.

Об'єкт дослідження – нестаціонарні хвилі сильних розривів, збурені в неоднорідному пружному середовищі.

Мета роботи – дослідити ефект взаємодії розривних хвиль з поверхнями розділу, які розмежовують анізотропні пружні середовища з відмінними фізичними властивостями.

Метод дослідження – метод нульового наближення променевого методу.

Розроблений метод ґрунтується на спільному використанні променевої теорії, локально-плоского підходу і теорії стереомеханічного удару. Він використовується для дослідження розповсюдження розривних хвиль в анізотропних тектонічних структурах. Дослідженні випадки взаємодії квазіпоздовжніх та квазіпоперечних хвиль з поверхнями розділу, які розмежовують анізотропні пружні середовища з відмінними фізичними властивостями. Розглянуті питання пов'язані з біфуркаціями хвильових фронтів і утворенням на них сингулярностей та каустик, а також концентрацій напружень і утворенням на них зон, в яких напруження прямують до нескінченності. Виконано моделювання ефектів екранування хвиль.

Результати статті можуть бути упровадженні в науково-дослідних і проектних організаціях, де займаються проектуванням споруд енергетичного, транспортного, гірничого призначення з метою їх сейсмозахисту та захисту від впливу розривних хвиль збурених техногенними чинниками.

КЛЮЧОВІ СЛОВА: ЕКРАНУВАННЯ, РОЗРИВНІ ХВИЛІ, АНІЗОТРОПНІ ШАРУВАТІ СЕРЕДОВИЩА, ПРОМЕНЕВИЙ МЕТОД.

ABSTRACT

Vashchilina O.V., Zaets Yu. O. Shielding discontinuous waves by anisotropic elastic layered medium. Visnyk National Transport University. – Kyiv. National Transport University. 2013. – Vol. 28.

The problem of dynamic interaction of wave phase fronts with anisotropic elastic media interfaces is considered.

Object of study – nonstationary waves of small discontinuities generated in nonhomogeneous elastic medium.

Purpose of the investigation –to investigate the effect of the interaction of discontinuous waves with interfaces separating anisotropic elastic media with different physical properties

Method of the study is the zero approximation of the ray method.

A technique based on joint use of the ray theory, locally plane approach and theory of stereomechanical impact is elaborated. It is employed for the investigation of discontinuous waves propagation in anisotropic tectonic structures. The cases of interaction of quasi-longitudinal and quasi-shear discontinuous waves with the interfaces separating different anisotropic elastic media are treated. The issues are considered which are associated with the wave front surfaces bifurcations, generation of their singularities and caustics, as well as with stress concentration and formation of zones where the stresses tend to infinity. The effects of the wave shielding are simulated. The analysis results are discussed.

The results of the article can be inculcated for elaboration of technologies of screening shock waves.

KEYWORDS: SHIELDING, DISCONTINUOUS WAVES, ANISOTROPIC LAYERED MEDIA, RAY METHOD.

РЕФЕРАТ

Ващилина Е.В. Экранирование разрывных волн слоистой упругой анизотропной средой. / Е.В. Ващилина, Ю.А. Заец // Вестник Национального транспортного университета. — К. : НТУ, 2013. — Вып. 28.

Рассмотрена проблема динамического взаимодействия волновых фазовых фронтов с поверхностями раздела анизотропных упругих сред.

Объект исследования – нестационарные волны сильных разрывов, возмущенные в неоднородной упругой среде.

Цель работы – исследовать эффект взаимодействия разрывных волн с поверхностями раздела, отделяющие анизотропные упругие среды с различными физическими свойствами.

Метод исследования – метод нулевого приближения лучевого метода.

В предложенной работе разработан метод, основанный на совместном использовании лучевой теории, локально-плоского подхода и теории стереомеханического удара. Он используется для исследования распространения разрывных волн в анизотропных тектонических структурах.

Исследованы случаи взаимодействия квазипродольных и квазипоперечных волн с поверхностями раздела, отделяющие анизотропные упругие среды с различными физическими свойствами. Рассмотрены вопросы связанные с бифуркациями волновых фронтов и образованием на них сингулярностей и каустик, а также концентраций напряжений и образованием на них зон в которых напряжения стремятся к бесконечности. Выполнено моделирование эффектов экранирования волн. Обсуждаются результаты анализа.

Результаты статьи могут быть внедрении в научно-исследовательских и проектных организациях, где занимаются проектированием сооружений энергетического, транспортного, горного назначения с целью их сейсмозащиты и защиты от влияния разрывных волн возмущенных техногенными факторами.

КЛЮЧЕВЫЕ СЛОВА: ЭКРАНИРОВАНИЕ, РАЗРЫВНЫЕ ВОЛНЫ, АНИЗОТРОПНЫЕ СЛОИСТЫЕ СРЕДЫ, ЛУЧЕВОЙ МЕТОД.

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